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Belief Propagation and Phase Transitions in Finite Connectivity Spin Glasses

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Recently, statistical mechanical approach has extensively been applied to various research fields beyond the natural science. Among them, probabilistic information processing, such as machine learning, error-correction code and so on, is a typical target and the replica method[1] originally developed in spin glass (SG) theory is efficiently used in these problems. It could be possible that an import from the information theory sheds a new insight into the statistical-mechanics problems.

Such an attempt was made in a recent work by Kabashima[2], in which an iterative inference algorithm called by the belief propagation(BP)[3] was applied to SG models. He found that in the fully connected SG model an instability condition of the fixed point in the BP iterative dynamics coincided with a thermodynamics instability, namely a phase transition. He also made a conjecture that the formal expression of the instability condition could be extended to a finite-connectivity mean-field SG defined on random graphs with a fixed coordination number. Then, the phase diagram on the parameter space of the temperature and the excess concentration of ferromagnetic bonds was obtained by choosing an appropriate fixed point in the expression. The obtained phase-boundary lines recover the previous results for the similar models obtained by the cavity method[4] and the effective field method[5], and furthermore include a non-trivial one which separates a ferromagnetic phase from a replica-symmetry-breaking one. The latter requires a complicated higher-order perturbations in other schemes like the cavity method and has not been obtained explicitly so far except for the zero-temperature limit[6].

The most interesting features of this scheme are the following points: (a) although the BP iterative dynamics does not have a direct link to the physical thermodynamics, its stability analysis gives thermodynamic properties and (b) the instability condition is expressed in a rather unified way, which is independent of a kind of the phase boundaries. Our motivation for the present work lies in the relationship between BP and the thermodynamic properties of the finite connectivity SG, which have attracted a renewed interest. In this work, we study two instability conditions of the BP dynamics and the thermodynamics of the model independently. The thermodynamic instability is here observed by using the Hessian analysis of the corresponding Bethe free energy around a phase of interest. The first condition associated with the BP dynamics is that the largest eigenvalue of the linearized stability matrix around a

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fixed point becomes unity as temperature decreases. The other condition for the thermodynamics is that the lowest eigenvalue of the Hessian matrix for a given minimum of the free energy becomes minimum or null at a certain critical temperature, which is expected to coincide with a thermodynamic phase transition temperature in a large-size limit. We numerically estimate these two temperatures in *finite-size* systems with a few different parameters of the SG model, where the ferromagnetic and SG transitions occur depending on the parameter. We have found that the two temperatures are significantly different from each other in the finite-size systems, and that they are seen to converge towards the phase boundary obtained in Ref. [2]. Our results suggest that the conjecture proposed in Ref. [2] does hold only in the thermodynamic limit. In my talk, I show our numerical results for the phase diagram of the finite connectivity SG in details and discuss a perspective of the present analysis in comparison with other schemes.

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