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Information geometrical view of propagation algorithms

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Pearl's belief propagation (BP) is a widely used algorithm to compute marginal distributions from a joint distribution defined with graphical models[9]. Generally, BP gives exact inferences only for distributions represented with tree graphs. However, we observe that BP gives good approximate inferences even for loopy graphs in many cases (loopy BP). Thus, BP is a very efficient algorithm to obtain a good approximate inference with small amount of computational cost.

Interestingly, we can find similar ideas of approximate inference in many fields. Bethe approximation and Kikuchi approximation in statistical physics are related to loopy BP. In error correction codes, low density parity check (LDPC) codes and turbo codes are known to be practical and powerful error correction codes, and their decoding algorithms are equivalent to loopy BP[5]. Moreover, it is pointed out that survey propagation[6] is also related to loopy BP[4], and we have revealed that adaptive TAP equations[8] and expectation consistent approximate inference[7] can be viewed as a family of loopy BP.

We have built a mathematical framework to analyze loopy BP[2, 3]. Based on the framework, we have studied the local convergence property, the accuracy of approximation, and its relation to so-called free energy based on information geometry[1]. In this talk, we would like to review the approximate inference methods listed above from information geometrical viewpoint.

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