Randomness and Computation
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Quantum Spin Glasses and Probabilistic Information Processing

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One of the most essential key points for probabilistic information processing is to choose the estimate $\overline{\sigma}_i$ of the original bit, which is degraded by some noise, by

$$\overline{\sigma}_i \quad = \quad \mathrm{sgn}[\langle \sigma_i \rangle]. \tag{1}$$

In this expression, the bracket $\langle \cdots \rangle$ means the average over the posterior given by the form of Gibbs distribution : $\sim \mathrm{e}^{-\mathcal{H}_{\mathrm{eff}}}$. In this framework, namely, so-called maximizer of posterior marginal estimation (the MPM estimation for short), many attempts have been done in the research field of probabilistic information processing. However, besides of this kind of *thermal fluctuation*, it might be possible to use *quantum fluctuation*.

For the simplest example, by adding the term : $-\Gamma \sum_i \hat{\sigma}_i^x$ to the effective Hamiltonian $\mathcal{H}_{\mathrm{eff}}$ (the Hamiltonian is now an operator as we will explain in following), the system is fluctuated due to *locally flipping probability* : $|\langle \sigma_i^z = \mp 1 | \hat{\sigma}_i^x | \sigma_i^z = \pm 1 \rangle|^2 = \Gamma^2$, where we defined operators :

$$\hat{\sigma}_i^x = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}, \quad \hat{\sigma}_i^z = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix} \tag{2}$$

and $\hat{\sigma}_i^z | \sigma_i^z = \pm 1 \rangle = \pm 1 | \sigma_i^z = \pm 1 \rangle$ with $|\sigma_i^z = +1\rangle = {}^t(1,0), |\sigma_i^z = -1\rangle = {}^t(0,1)$. Therefore, a sequence of the messages/pixels is now represented by $|\{\sigma^z\}\rangle = |\sigma_1^z\rangle \otimes \cdots \otimes |\sigma_N^z\rangle$ with each message/pixel $|\sigma_i^z\rangle = \alpha |\sigma_i^z = +1\rangle + \gamma |\sigma_i^z = -1\rangle$.

Recently, Tanaka and Horiguchi [1] introduced this kind of fluctuation, instead of the thermal one, into the mean-field annealing algorithm and showed that performance of the image recovery is improved by controlling the amplitude Γ appropriately during its annealing process. However, there are no yet analytical studies concerning about the properties of the quantum MPM estimation, *especially at zero temperature* (the quantum MPM estimation at temperature 1 was already investigated by the present author [2]).

In this talk, I will make this point clear and the best possible performance of the quantum MPM estimation is compared with that of the thermal MPM estimation. In following, we show the outline of several results.

Example 1 : Image restoration

For image restoration, the effective Hamiltonian (the shoulder of exponential in the posterior) is given by

$$\mathcal{H}_{\mathrm{eff}}^{Quantum} \quad = \quad -\beta_m \sum_{<ij>} \hat{\sigma}_i^z \hat{\sigma}_j^z - h \sum_i \tau_i \hat{\sigma}_i^z - \Gamma \sum_i \hat{\sigma}_i^x. \tag{3}$$

In Figure 1 (left), we plot the bit-error rate for the quantum MPM estimation and the thermal MPM estimation. We find that the performance of the quantum MPM estimation is superior to the MAP estimation ($T_m, \Gamma_{\mathrm{eff}} \equiv \Gamma/\beta_m \to 0$) and there exists some finite value of the amplitude $\Gamma_{\mathrm{eff}} = \Gamma_{\mathrm{eff}}^{\mathrm{opt}}$ at which the bit-error rate takes its minimum. We also obtain the Nishimori-Wong condition [3] which gives the condition of best possible choice of the hyper-parameters for the MPM estimation :

$$\textbf{Thermal :} \; T_m^{\mathrm{opt}} = T_s \quad \text{(Nishimori and Wong [3])}$$
$$\textbf{Quantum :} \; m_0(\beta_s) = \sum_{\xi} \mathcal{M}(\xi) \int_{-\infty}^{\infty} \frac{\phi_0 Du}{\sqrt{\phi_0^2 + (\Gamma_{\mathrm{eff}}^{\mathrm{opt}})^2}}$$

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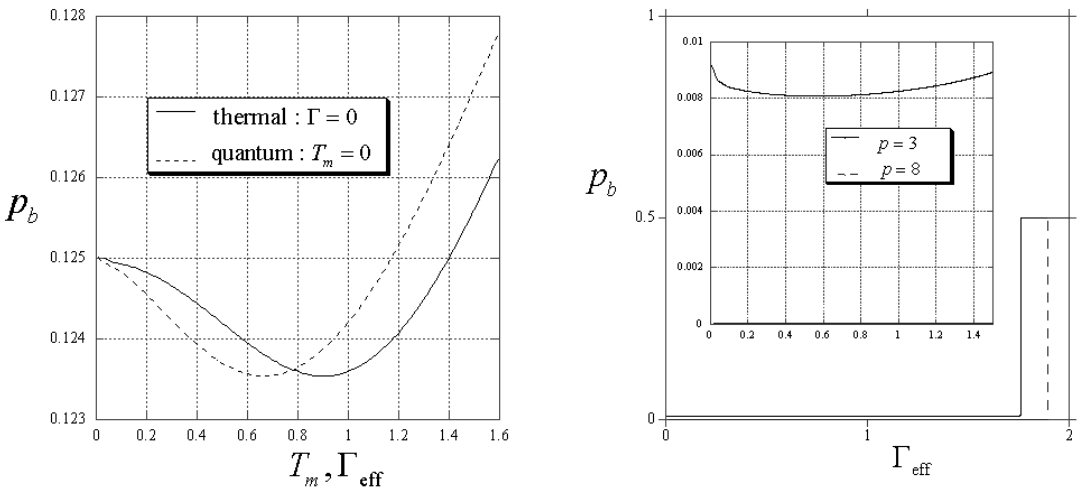

Figure 1: The bit-error rate for the quantum ($T_m = 0$) and the thermal (classical) ($\Gamma = 0$) estimation (left). The right panel shows The Γ_{eff}-dependence of the bit-error rate for the Quantum Sourlas codes with $p = 3$ and 8.

where $T_s = \beta_s^{-1}$ is true temperature of the ferromagnetic Ising model for the original image and m_0 is a solution of $m_0 = \tanh(\beta_s m_0)$. We also defined : $\mathcal{M}(\xi) = \mathrm{e}^{\beta_s m_0 \xi}/2\cosh(\beta_s m_0)$, $\phi = m + h_* a_0 \xi + h_* a u$, $Du = du\, \mathrm{e}^{-u^2/2}/\sqrt{2\pi}$ and a_0/a is signal to noise ratio of the channel.

Example 2 : Sourlas error-correcting codes

The quantum version of the effective Hamiltonian for Sourlas error-correcting codes [4] is given by

$$\mathcal{H}_{\mathrm{eff}}^{Quantum} = -\beta_J \sum_{i1\cdots ip} J_{i1\cdots ip}\, \hat{\sigma}_{i1}^{z} \cdots \hat{\sigma}_{ip}^{z} - \Gamma \sum_{i} \hat{\sigma}_{i}^{x}. \tag{4}$$

In Figure 1(the right panel), we plot the $\Gamma_{\mathrm{eff}}(\equiv \Gamma/\beta_J)$-dependence of the bit-error rate for the case of $p = 3$ and 8. In this figure, we find that the optimal amplitude of the transverse field $\Gamma_{\mathrm{eff}}^{\mathrm{opt}}$ exists (see the inset), but beyond the critical transverse field $\Gamma_{\mathrm{eff}}^{\mathrm{c}}$, the system undergoes first-order phase transition from the state $P_b \simeq 0$ to the state $P_b = 1/2$.

The details are reported in the workshop.

References

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