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Generation of complex networks without growth

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As for many complex systems, network structures are important as their backbone. From research on dynamics of epidemic and disease spreading, it has been revealed that network structures have large influence on spreading phenomena, such as computer virus infections, epidemiology, and rumor dynamics [1]. The network structures affect various dynamics on the networks, and conversely various dynamics might affect the network structures and could change them. Therefore research on network structures is important, and then research on complex networks has attracted the attention of a lot of scientists [2, 3]. Recently, investigations on a wide variety of networks, such as World Wide Web [4], the Internet backbone [5], social networks [6], the metabolic networks [7], and so on have clarified that complex networks in nature are *more complex* than ordinary random networks proposed by Erdős and Renýi [2, 8]. One of the remarkable properties of the real complex networks is a *scale-free property*. If we define the degree distribution $P(k)$ as the probability that a node is connected to k other nodes, networks with the scale-free property are characterized by a power-law behavior, $P(k) \sim k^{-\gamma}$, where γ is a characteristic degree exponent.

Barabási and Albert [9] have proposed a model (so-called BA model) with the concepts of *growth* and *preferential attachment* in order to explain emergence of the scale-free networks in nature. The concept of *growth* means that a network get a new node with time, and that of *preferential attachment* means that the newly added node is linked to old nodes with a probability proportional to the degree of those old nodes. The probability used in the preferential attachment is as follows:

$$\Pi_i = \frac{k_i}{\sum_j k_j}, \tag{1}$$

where k_i is the number of edges connected to node i. It has been shown that the BA model makes a scale-free network with a characteristic degree exponent $\gamma = 3$ [2].

While it has become clear that the two concepts (growth and preferential attachment) generate complex networks with the scale-free property, it is still unclear what is essential for generating complex networks with the scale-free property. It is an open question that the growth property is suitable in order to explain emergence of the scale-free networks. A certain class of real networks are described by growing networks; in networks of citations in scientific paper, nodes denote scientific papers and edges represent citations. When a new paper is written, a node is added to the network and newly added edges are connected according to the citations in the new paper. In the citation networks, once edges are linked between papers, those edges in the network are fixed and never removed. However, some networks in nature can experience their structural changes that are relatively much faster than network growth. For instance, in economical networks of companies, friendship networks, peer-to-peer (P2P) networks, and so on, it seems natural that the number of agents (such as companies, human) increases or decreases slowly and hence is considered to be fixed approximately, while connections or relationships between agents change in turn rather rapidly. Then, one would consider that their networks are not growing approximately, but there are a lot of rewirings of relationships; this is a kind of adiabatic approximations.

We here classify models for generating complex networks as follows:

(1) "Growing networks": the numbers of nodes and edges are increasing with time.

(2) "Quasi-static networks": the number of nodes is fixed but the number of edges grows with time.

(3) "Static networks": the numbers of nodes and edges remain fixed and do not grow with time. Additionally, the network structure is never changed with time.

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(4) "Non-growing networks": the numbers of nodes and edges are fixed, but arrangement of the nodes and/or edges can occur dynamically by some rules.

There are a lot of models for growing networks [2]. Recently, some of scientists have focused their attention to static or non-growing models for complex networks, and several models have been proposed in order to generate static or non-growing complex networks with the scale-free property [10–15].

We here propose a non-growing model for generating complex networks. The detail of the model is written in ref. [14]. The model uses only preferential rewiring processes with fitness parameters. We start with a classical random network, and at each time step, we apply the following operations to the initial network:

(i) We select a node i and an edge $\hat{l}_{ij}$ connected to it randomly with a uniform distribution.

(ii) We remove the edge $\hat{l}_{ij}$, and replace it with a new edge $\hat{l}_{ij'}$ that connects node i with node j'. Node j' is chosen randomly with a probability

$$\Pi_{j'} = \frac{\eta_{j'}(k_{j'}+1)}{\sum_j \eta_j (k_j+1)}, \tag{2}$$

where k_j is the number of edges connected to node j. Here, if the edge $\hat{l}_{ij'}$ already exists, we choose a different node j'' with the probability $\Pi_{j''}$. A coefficient η_j is a fitness parameter, which represents that each node has different ability to compete for edges. A value of the fitness parameter η_j is chosen from a fitness distribution $\rho(\eta)$. We assume that once the fitness parameter η_j is assigned to each node i, it does not change in time.

Using the operations (i) and (ii) repeatedly, the proposed model generate complex networks different from the ordinary random networks. Moreover, some of the obtained networks show a scale-free-like property by using suitable fitness distributions. However, when we set the fitness distribution $\rho(\eta)$ as a constant fitness distribution, $\rho(\eta) = \delta(\eta - 1)$, the obtained networks do not show the scale-free property. Then, it is revealed that the preferential attachment mechanism alone dose not give scale-free networks in the non-growing model, and one more factor is necessary in order to generate networks with the scale-free property. In our model, the additional factor is the fitness parameter. It could be important to study complex networks with randomness in which each node (agent or element) has different property or ability.

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