Genetic Algorithms for Optimization of Noisy Fitness Functions and Adaptation to Changing Environments

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1 Introduction

Genetic Algorithms (GAs) are optimization, adaptation and learning algorithms inspired by the natural selection theory of evolution. GAs attract attention as methods not only for conventional optimization problems, but also for optimization of uncertain functions because of their characteristics of

- Direct optimization method that uses only fitness function values,
- Stochastic search for global optimization, and
- Self-averaging nature of population-based search.

There have been proposed many variations of GAs for optimization of uncertain functions, e.g, optimization of noisy fitness function and adaptation to changing environments.

Promising applications of GAs for optimization of uncertain functions are

- *Online adaptation* of the systems working in the real world, especially, systems that face difficulty in constructing their precise simulators. In such systems, some design parameters should be decided through experiments, and therefore good ways for optimization through experiments are needed. Further, the optimal parameters may change by the condition of usage, and in such a case, online adaptation of the parameters is preferred.
- *Simulation-based optimization* of large and complex systems that uses random numbers in the simulation. Examples are optimization of traffic, communication and production systems, which is carried out using evaluation by discrete event simulation. Random numbers are used for generation of demands, faults and other events in the simulation. It causes fluctuation of the fitness values obtained by simulation.

It should be noted that available numbers of evaluation of fitness functions are severely restricted in most of such applications. Hence, GAs considering not only uncertainty of the fitness functions but also restriction in fitness evaluation should be developed.

Considering the aforesaid requirement, the authors have developed the memory-based fitness evaluation GA (MFEGA)[1, 2, 3] for optimization of noisy fitness functions and the GA using sub-population (GASP)[4] for adaptation to changing environments. These GAs have been successfully applied to practical problems. The following sections give an overview of these GAs and their applications. This paper is organized as follows: In Section 2, the problem of optimization of noisy fitness functions is formulated and the previous approaches in GAs to this problem are reviewed. In Section 3, the concept, algorithm and applications of the MFEGA are described. In Section 4, the problem of adaptation to changing environments and the previous approaches in GA to this problem are introduced. In Section 5, a class of such problem is formulated and GAPS developed for this class is described. Section 6 concludes this paper.

2 Optimization of Noisy Fitness Functions

2.1 Formulation of the Problem

One sorts of uncertainty in systems to be optimized is noise involved in measurement of performance of the system. We call such problem 'optimization of noisy fitness functions,' and formulate the problem as follows

$$\min_x \langle F(x) \rangle \tag{1}$$
$$F(x) = f(x) + \delta \tag{2}$$

where x is the decision variable, $F(x)$ is the observation of fitness value, $f(x)$ is the true fitness function, δ is additive noise and $\langle\rangle$ denotes expectation over δ. We assume that x is continuous variable, and $\langle \delta \rangle = 0$.

A related problem is '*robust optimization of the system*.' It aims to find a robust solution that works well even if the system deviates from its nominal model. One formulation of robust optimization will be

$$\min_x \langle f(x + \delta) \rangle \tag{3}$$

where x is the decision variable, $f(x)$ is the nominal fitness function, δ is random deviation of the system from the nominal model. In the above formulation, deviation δ appears additively to the decision variables, which is a model of implementation error of decision variable. While the random deviation δ is assumption introduced by the designer, the Monte Carlo approach using random number for δ is often taken because of difficulty in obtaining expectation over multi-dimensional deviation δ. Hence, this problem is very similar to the problem of noisy fitness function.

2.2 GA Approaches to Noisy Fitness

Optimization of noisy fitness functions, i.e., to find a point having the maximum (or minimum) mean value of the fitness function observed with random fluctuation, is one of the important applications of the GAs. While optimization of noisy function is also discussed in other contexts, e.g., in the stochastic approximation algorithms[11], it is expected that GAs can find a good solution of the problems because of their global search ability, and flexibility in implementation that makes introduction of domain knowledge easy.

GA approaches to the problem of optimization of noisy fitness function proposed so far can be categorized into the following three types.

Application of Conventional GAs The population based search by GAs has self-averaging nature[5]. That is, solution having good fitness in average survives as population. Hence, simply applying conventional GAs can be a method for the problem. However, it requires large population sizes, and takes long time for convergence. Hence, more sophisticated approach is needed.

GA with Multiple Sampling Another approach to cope with noisy fitness is to sample fitness values several times for each individual, and use the mean of the sampled values for evaluation of the individual[6]. This approach reduces the variance of fitness values without any assumptions on fitness function. However, it requires large number of evaluations to reduce the variance sufficiently. Considering practical applications such as optimization through experiments or complicated simulation, it is a serious drawback in this approach. There have also proposed methods to improve this approach[7, 8].

Referring to Fitness Values of Other Individuals A third approach is to evaluate an individual not only by its sampled fitness value but also by those of other individuals near it. This technique requires some assumption on the fitness function that fitness value of nearby solution gives some information of the fitness value at the point of interest. Usually it is assumed implicitly that nearby solutions take similar fitness values with that at the point of interest.

Tamaki et al. and Tanooka et al. have proposed a technique which refer to the fitness value of a parent[9, 10]. However, since the parents are survivors in selection, its fitness value has some bias toward apparently better values. Hence the estimated value involves systematic error. Branke has proposed a method which refer to the sampled values of the nearby individuals in current and previous generation[7]. In this method, the number of the samples used for estimation is limited. Memory-based Fitness Evaluation GA (MFEGA) proposed by the authors[1, 2] also belongs to this category. The detail of the MFEGA is described in the next section.

3 Memory-based Fitness Evaluation GA

3.1 Concept of MFEGA

So as to optimize noisy fitness functions within a restricted number of fitness evaluation, the authors have proposed the Memory-based Fitness Evaluation GA (MFEGA)[3]. The key ideas of the MFEGA are as follows:

- To store sampled fitness values into memory as search history.
- To introduce a simple stochastic model of fitness values for estimation.
- To estimate fitness values of points of interests using the history for selection operation in GA.

3.2 A Stochastic Model of Fitness Functions

MFEGA adopts a simple stochastic model of fitness that fitness values of individuals distributed randomly around the fitness value at the point of interest, and has assumed that the variance of the fitness value depends only on the distance from the point of interest.

Let $\boldsymbol{x}$ be an individual we want to estimate its fitness value, and let $\boldsymbol{h}$ be an individual in the history of search whose distance from $\boldsymbol{x}$ is d, and a sampled fitness value for $\boldsymbol{h}$ is $F(\boldsymbol{h})$. On the distribution of $F(\boldsymbol{x})$, the following model is assumed:

$$f(\boldsymbol{h}) \sim N(f(\boldsymbol{x}), kd) \tag{4}$$
$$\delta \sim N(0, \sigma^2) \tag{5}$$
$$F(\boldsymbol{h}) = f(\boldsymbol{h}) + \delta \sim N(f(\boldsymbol{x}), kd + \sigma^2) \tag{6}$$

where $f(\boldsymbol{h})$ is the true fitness of individual $\boldsymbol{h}$, and k is a positive parameter. The additive noise δ is assumed to be Gaussian. As shown in Fig.1, Eqs. (4) through (6) mean that the true fitness $f(\boldsymbol{h})$ distributed randomly around the $f(\boldsymbol{x})$ with variance proportional to the distance d, and hence observation of the fitness $F(\boldsymbol{h})$ follows the normal distribution of a variance $kd + \sigma^2$ considering the additive observation noise.

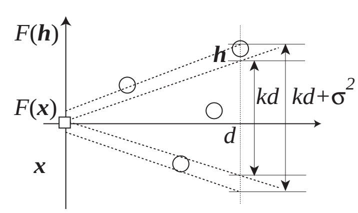


Figure 1: Stochastic model of noisy fitness.

3.3 Maximum Likelihood Estimation of Fitness

Assuming that parameters k and σ^2 are known in advance, the fitness value $f(\boldsymbol{x})$ can be estimated by the maximum likelihood method using the search history based on the above model.

Let $\boldsymbol{h}_l, l = 1, ..., H$ be the H individuals in the search history and $F(\boldsymbol{h}_l)$ and $d_l, l = 1, 2, ..., H$ be their sampled fitness values and distances from the individual of interest, respectively. The probability of obtaining $F(\boldsymbol{h}_1), ..., F(\boldsymbol{h}_H)$ is represented by

$$\prod_{l=1}^{H} p(F(\boldsymbol{h}_l), d_l) \tag{7}$$

where $p(F(\boldsymbol{h}_l), d_l)$ is the probability density function of $F(\boldsymbol{h}_l)$ given by:

$$p(F(\boldsymbol{h}_l), d_l) = \frac{1}{\sqrt{2\pi(k'd_l + 1)\sigma^2}} \exp\left(-\frac{1}{2}\frac{(F(\boldsymbol{h}_l) - f(\boldsymbol{x}))^2}{(k'd_l + 1)\sigma^2}\right) \tag{8}$$

where $k' = k/\sigma^2$.

With the maximum likelihood technique, estimation of $f(\boldsymbol{x})$, say $\tilde{f}(\boldsymbol{x})$, can be obtained as a weighted average of sampled fitness values:

$$\tilde{f}(\boldsymbol{x}) = \frac{F(\boldsymbol{x}) + \sum_{l=2}^{H} \frac{1}{(k'd_l + 1)} F(\boldsymbol{h}_l)}{1 + \sum_{l=2}^{H} \frac{1}{(k'd_l + 1)}} \tag{9}$$

3.4 Estimation of the Model Parameters

In actual optimization, the parameters k' in Eq. (9) is not known in advance. Hence it is needed to estimate this parameter. We also employ the maximum likelihood technique for estimation of k' and σ^2. Taking Eq. (7) as the likelihood of the parameters k' and σ^2, and a logarithm likelihood of the parameters is calculated from Eq. (7) as follows:

$$\begin{aligned} \log L &= -\frac{1}{2}\left(H \log 2\pi + \sum_{l=1}^{H}(k'd_l + 1)\sigma^2 \right. \\ &+ \left. \sum_{l}^{H} \frac{(F(\boldsymbol{h}_l) - f(\boldsymbol{x}))^2}{(k'd_l + 1)\sigma^2}\right) \end{aligned} \tag{10}$$

Since this equation also includes unknown variable $f(\boldsymbol{x})$ and d_l, we set $\boldsymbol{x}$ at the individual which has the smallest sampled fitness considering that the usage of the model is optimization of $f(\boldsymbol{x})$. The fitness value of $f(\boldsymbol{x})$ is simply estimated by the average of the sampled fitness values of five individuals near by. Distance d_l is easily calculated if $\boldsymbol{x}$ is decided. Considering positiveness of the parameters, a numerical hill climbing method w.r.t. logarithms of k' and σ^2 is used for maximization of the likelihood.

3.5 Prototype Algorithm of the MFEGA

The following is a prototype algorithm of the MFEGA. For selection operation, we employ a steady state type one called MGG[14]. For crossover operator, we use the unimodal normal distribution crossover (UNDX) proposed by Ono et al. [13] considering application to optimization in continuous search spaces. Since the UNDX has excellent search ability, no mutation is used.

(*Initialization*)

1. Initialize the population of M individuals $\boldsymbol{x}_1, ..., \boldsymbol{x}_M$ randomly.
2. Let evaluation counter $e = 0$. Set the maximal number of evaluations to E.
3. Let history $H = \phi$.

(*Main Loop*)

4. Choose two individuals $\boldsymbol{x}_{p1}$ and $\boldsymbol{x}_{p2}$ from the population as a pair of parents.
5. Produce C children $\boldsymbol{x}_1^c, ..., \boldsymbol{x}_C^c$ by applying the crossover to the parents.
6. Let $\boldsymbol{y}_1 = \boldsymbol{x}_{p1}$, $\boldsymbol{y}_2 = \boldsymbol{x}_{p2}$ and $\boldsymbol{y}_{i+2} = \boldsymbol{x}_i^c, i = 1, ..., C$. We call the set $\{\boldsymbol{y}_1, \cdots, \boldsymbol{y}_{C+2}\}$ a family.
7. Sample fitness value F for $\boldsymbol{y}_i, i = 1, ..., C+2$.
8. Let $e = e + C + 2$.
9. Store the sampled values into the history H, i.e, $H = H \cup \{(\boldsymbol{y}_i, F(\boldsymbol{y}_i)) | i = 1, ..., C+2\}$.
10. Select the individual $\boldsymbol{h}_{min}$ having the smallest sampled fitness value from H.
11. Estimate k' and σ^2 by maximization of Eq. (10).
12. Estimate $f(\boldsymbol{y}_i)$ by Eq. (9).
13. Substitute the individual having two smallest $\tilde{f}(\boldsymbol{y}_i)$ among $\{\boldsymbol{y}_1, ..., \boldsymbol{y}_{C+2}\}$ into $\boldsymbol{x}_{p1}, \boldsymbol{x}_{p2}$
14. If $e \leq E$, go to Step 4, otherwise terminate the algorithm.

3.6 Numerical Examples

The performance of the proposed method is compared with conventional methods through numerical experiments. For the test function, the following 10 dimensional sphere function with additive noise is used:

$$
\begin{aligned}
F_1(\boldsymbol{x}) &= \sum_{j=1}^{10} x_j^2 + \delta, \\
& \quad \delta \sim N(0, \sigma_{F_1}^2), \quad \sigma_{F_1}^2 = 1.0,
\end{aligned} \tag{11}
$$

where $\boldsymbol{x} = \{x_1, x_2, ..., x_{10}\}^T$ is a decision vector, and δ is an additive normal noise . Initial population is sampled in $[-0.5, 0.5]^{10}$ randomly.

The following three algorithms are compared.

- Standard GA: A GA using a single fitness sample for evaluation of an individual.
- Sample 10-GA: A GA using the mean of 10 fitness samples for evaluation of each individual.
- MFEGA.

Same operators and parameters are employed except for estimation techniques of the fitness value. The population size M is 30, and the children size C is 5. These parameters are chosen considering application of our method to the practical control problem[12]. The number of maximum fitness evaluation is set at 10,000.

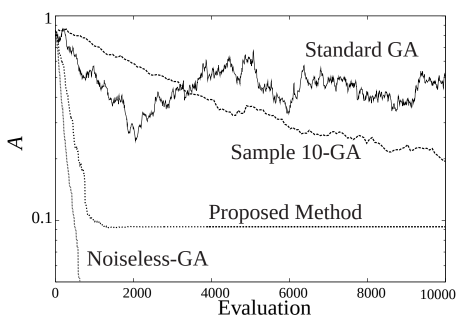
Performance of the tested methods is evaluated from two points of view. That is, how closely each algorithm converges to the optimum, and how accurately fitness values are estimated in each algorithm. From these viewpoints, indexes A and D are introduced respectively:

$$
A = \frac{1}{M} \sum_{m=1}^{M} f(\boldsymbol{x}_m), \tag{12}
$$

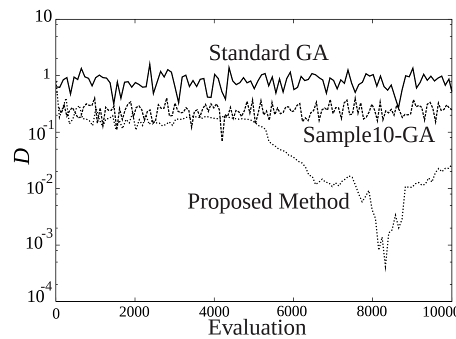
$$
D = \frac{1}{P+C} \sum_{i=1}^{P+C} |\tilde{f}(\boldsymbol{x}_i) - f(\boldsymbol{x}_i)| \tag{13}
$$

where A is the mean of the true fitness values of the individuals in the population, and D is the average of the deviation of the estimated fitness values from true ones. Figure 2 (a) shows the evolution of index A. In the standard GA, reduction of index A stagnates after around 2000 evaluations, and the value of A fluctuates around 0.3. Contrary to this, in the sample 10-GA exhausts all the evaluations before convergence while it succeeded in finding better solutions. The proposed method converges quickly and finds more accurate solutions than the other two methods. In Fig. 2 (a) result of GA applied to the fitness function without noise (the noiseless-GA) is also plotted. The proposed method achieve a convergence speed about half of the noiseless GA in early stage of the search.

Figure 2 (b) shows evolution of the index D. We can confirm that D of sample 10-GA achieves better estimation than the standard GA as predicted by the theory. Concerning the proposed method, it achieve estimation of almost same precision with Sample 10-GA up to 5000 fitness evolution and it gets better later.



(a) Search precision A for F_1



(b) Estimated precision D for F_1

Figure 2: Results of the Numerical experiments with the MFEGA

3.7 Tested-MFEGA

While MFEGA works well for interpolative search, the process of GA faces difficulties of stagnation in extrapolative search because the estimation of the fitness function loses its effectiveness in such case. To cope with this problem, the authors have also proposed 'the tested-MFEGA' that combines MFEGA and z-test[4]. In the tested-MFEGA, after sampling of fitness values for the family, find a individual, say the best individual, that has the best sampled fitness value, and members having sufficiently poor sampled fitness values in the family are excluded from the candidates for survivor. Numerical experiments show that the tested-MFEGA is slightly slower in convergence speed in earlier stage of search than the original MFEGA, but it steadily converges avoiding stagnation.

3.8 Applications of MFEGA

Online Adaptation of Vehicle Engine Sano et al. have applied the MFEGA to the problem of vehicle engine control[2]. The problem is to improve the response of the engine by dynamic control of the air/fuel ratio[12]. For this study, a computer simulator considering dynamics of air and fuel in the intake manifold and the cylinder is used instead of real experiments. Fitness function can be observed as response of the engine for acceleration manipulation of the throttle.

In the real system, observed fitness value is affected by implementation of the acceleration input, condition of the road, and other many factors. Since the simulator provides a noise free environment, observation noise is added artificially. The variance of the noise is decided considering fluctuation observed in experiments with the real engine. The results of numerical simulation shows that the MFEGA finds good control policy within permissible number (around several hundreds) of fitness evaluation.

Simulation-based Optimization of Controller for Multi-Car Elevators The MFEGA is also applied to simulation-based optimization of multi-car elevator (MCE) controller[15]. Multi-car elevator is a elevator system that has several cars in a single elevator shaft. It is a novel elevator system under development with background of progress in linear motor technology. However, since applicability of knowledge of the conventional group control of elevators to the MCE is limited, control schemes for MCE should be constructed from scratch. Simulation-based optimization is one of the promising approach in design of the MCE controller. However, simulation is discrete event type using random numbers, and it takes long computation time. Takahashi et al. has applied MFEGA to optimization of MCE using a PC cluster to accelerate computation time by parallel evaluation of individuals in GA. MFEGA shows better solution quality compared with the conventional GA, and shorter computation time compared with multiple-sampling GA.

4 Adaptation to Changing Environments

Another uncertainty in optimization of system is adaptation to changing environments. That is, the fitness function to be optimized changes over time due to environmental change. For this category of problem, there have also been proposed many variation of GAs.

In consideration of adaptation to changing environments, since loss of the diversity of population reduces the adaptation ability of the GA, maintenance of the diversity is an essential requirement in such applications. Several studies have been reported on GAs for solving non-stationary optimization problems. Grefenstette[17], Gobb and Grefenstette[18] have been proposed methods of controlling the mutation rate. Mori et al. have proposed to utilize the thermodynamical genetic algorithm (TDGA) to such problems[19, 20]. TDGA is a genetic algorithm which evaluates the diversity of the population by *entropy*, and selects the population so as to minimize the *free energy*. We call these methods *the search-based approach*[21].

If the environmental change is recurrent, memorizing the results of past adaptations and utilizing them as candidates for the solutions will be an effective strategy. We call such an approach *the memory-based approach*[21]. In the context of GA research, several studies in this approach have been proposed[16, 22]. In these studies, results of past adaptation are memorized by introducing redundant genetic representation such as diploidy. A more explicit usage of memory is proposed by K. Mori et al. [23]. It is an algorithm inspired by the immune system, and called 'the immune algorithm'[23]. From the viewpoint of artificial intelligence, the immune algorithm is combination of GA and memory-based reasoning. Mori et al. has also proposed a method in memory-based approach called the Memory-Based TDGA[24].

In these proposal, however, the formulated problems are abstract and conceptual, and the proposed methods are discussed only based on such formulation without consideration of practical applications. Since the problems of adaptation to changing environment are by themselves hard problems, a more specific formulation of the problem based on practical application, and development of more effective methods based on the formulated problem are needed.

5 Genetic Algorithm using Sub-Populations

5.1 A Model of Changing Environment

Considering the aforesaid difficulties in adaptation to changing environments, the authors have started discussion from a practical application. It is a problem of engine control of motor boat. Depending on the mode of operation such as 'to go straight', 'to slalom', and 'to turn', the dynamics of the boat changes largely and hence it is needed to change control scheme for engine to keep the performance of engine, e.g., the speed of the boat. It can be formulated as a problem of optimization of several fitness functions randomly switched corresponding to the operation modes. Figure 3 illustrates the model. Nodes correspond modes of operation, and arcs represents transition among the modes.

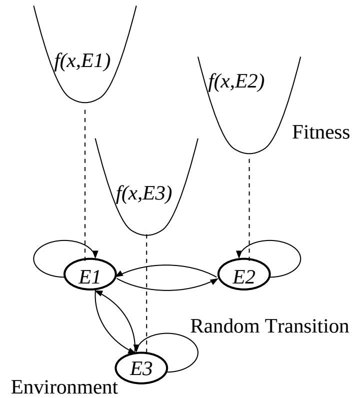


Figure 3: Model of changing environment.

5.2 Control and Optimization Scheme in the Changing Environment

Control and its optimization is carried out with the process illustrated in Fig. 4. Before applying a control scheme, it is possible to estimate which mode will continue for a while by observing some variables of the system. We call it 'the prior estimation'. Based on the prior estimation, a control scheme for the estimated environment is selected and applied. After, operation of the system for a prescribed duration, we obtain the performance evaluation of the applied control scheme. At the same time, we can estimate the occurred mode. We call it 'the posterior estimation'. In general, the posterior estimation will be more precise than the prior estimation. Using the obtained observation of performance and the posterior estimation, the control scheme is updated.

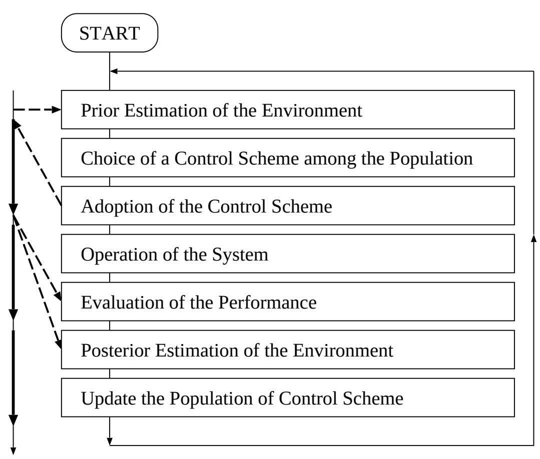


Figure 4: Control scheme in changing environment.

5.3 Prototype Algorithm of GASP

For this problem, the authors have developed a Genetic Algorithm using Sub-Populations (GASP)[4]. In GASP, for each appearing mode, a corresponding sub-populations corresponding is prepared, and adaptation will be carried out by repeating the following process:

1. Initialize individuals in each sub-population, and for each sub-population, prepare a family for evaluation by selecting parents and applying crossover to them.

2. Obtain a prior estimation based on observation of the system.

3. Apply a control scheme belong to the family of sub-population corresponding to the prior estimation.
4. Operate the system for a prescribed duration.
5. Obtain evaluation of the applied control scheme.
6. Obtain a posterior estimation based on observation of the system.
7. Put the evaluated control scheme into the family of a sub-population corresponding the the posterior estimation.
8. If all the members in the family are evaluated, apply selection operation using MGG to the corresponding sub-population, and prepare a new family for the sub-population.
9. Go to Step 2 until termination criterion holds.

The GASP can be successfully applied to a problem of switching environment which is formulated based on the observation in the experiments using a real motor boat.

6 Conclusion

In this paper, genetic algorithms for optimization of uncertain function is discussed. The authors have proposed the Memory-based Fitness Evaluation GA (MFEGA) for the problem of noisy fitness function, and the GA using Sub-Population (GASP) for the problem of adaptation to changing environments. While the both GAs were developed separately, the noise involved in the fitness also appears in the problem of adaptation to changing environments. Hence, integration of the MFEGA and GASP is a subject of future study.

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